Te Poutāhū
Curriculum Centre

Te Mātaiaho

THE REFRESHED NEW ZEALAND CURRICULUM

The learning areas of Mātaiaho

Mathematics and statistics

Mātai aho tāhūnui,
Mātai aho tāhūroa,
Hei takapau wānanga
E hora nei.
Lay the kaupapa down
And sustain it,
The learning here
Laid out before us.

Te Tāhuhu o te Mātauranga
Ministry of Education

Te Kāwanatanga o Aotearoa
New Zealand Government

Purpose statement for mathematics and statistics

Ānō me he whare pūngāwerewere.
Behold, it is like the web of a spider.

This whakataukī celebrates intricacy, complexity, interconnectedness, and strength. The learning area of mathematics and statistics weaves together the effort and creativity of many cultures that over time have used mathematical and statistical ideas to understand their world.

Mathematics and statistics enables students to appreciate and draw on the power of abstraction and symbolic representation to investigate, interpret, and explain patterns and relationships in quantity, space, time, data, and uncertainty. Like mathematics and statistics, mātauranga Māori is a body of knowledge with a history and a future. When we afford mana ōrite to mātauranga mathematics and statistics and mātauranga Māori while retaining their distinctiveness, students can draw from both in ways that are beneficial to both spheres of knowledge.

The learning area has been designed to support the vision of Mātaitipu and provides personal value, participatory value, pathways value, and planetary value. Collectively, these show the richness and value of mathematics and statistics learning for students. Students discover inherent personal enjoyment and satisfaction in persistence, solving problems, identifying patterns, and seeing the beauty in mathematics and statistics. They come to appreciate the everyday use of mathematical and statistical tools in, for example, personal finance, music and dance, estimation, and measurement. They recognise how their culture is included and valued in the learning area.

Students participate as they take part in discussions with their peers about their mathematical and statistical thinking and the thinking of others. They discuss and take action on important social matters such as the ethical gathering, interpretation, and communication of data, and challenging misinformation and disinformation. They also engage with diverse cultural perspectives, including te ao Māori and Pacific world-views, on being numerate in Aotearoa New Zealand.

Through the learning area, students can discover pathways into a wide range of industries that rely on mathematical knowledge and reasoning. This allows them to participate fully in an increasingly technology- and information-rich world of work. Learning in mathematics and statistics is important for realising the aspirations and priorities of every student and their parents and whānau.

Students also come to understand the value of mathematical and statistical modelling as a lens for resolving collective global challenges – for example, in adapting to and mitigating climate change and in helping to build an equitable, sustainable future for all.

Learning in mathematics and statistics builds both literacy and numeracy. Mathematics and statistics contribute to students' literacy by developing their skills in oral and written communication, meaning-making, and the use of specific vocabulary and symbols. Statistics and probability, in particular, support the understanding of tables, graphs, and diagrams as well as critical thinking about the quality of data and stories told about it.

As this whakataukī tells us, connections between different concepts, knowledge, and practices are central to mathematics and statistics. Teachers weave together the elements of Understand, Know, and Do to ensure students learn mathematics and statistics as a connected body of knowledge.

Planning for teaching

The most effective teaching of mathematics and statistics follows a strengths-based approach that creates opportunities for all students to learn and progress. Such an approach recognises that all students exist within their whānau and culture and includes parity for mātauranga Māori.

It is important for teaching to be ambitious within and potentially beyond each phase of learning. When designing a mathematics and statistics programme, teachers need to plan for providing students with multiple opportunities to progress.

Learning happens best when mathematics and statistics are taught daily, using purposeful tasks related to both mathematical contexts and wider contexts relevant to the communities, cultures, interests, and aspirations of students.

When planning how to support progress, teachers can ask: What opportunities do students have to:

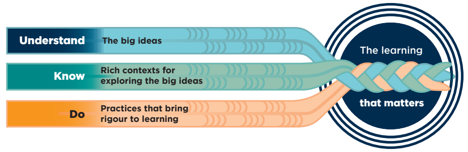
- learn new mathematics and statistics concepts and practices?
- use mathematics and statistics to investigate relevant tasks?
- communicate and critique mathematical findings and understandings?
- understand the interrelated nature of skills and concepts in mathematics and statistics?
- practise the mathematics and statistics that they have learned?

When planning tasks, teachers can ask:

- What are the cultural contexts that will resonate with my students?
- How can I support students to engage with a context's whakapapa, tikanga, and significance while honouring and maintaining the integrity of both the mathematics and the context?

As they prepare, teachers can work through the tasks themselves and ask:

- How can I help students find the joy in this learning?
- How can I help build the resilience of students?
- How can I value and reward persistence?
- How can I help students see the broad relevance of this work to their lives, including purposeful contexts, mathematical skills, social skills, knowledge, cognitive development, and cultural competence?



Understand

Big ideas

> ***Whiria te kaha tūātinitini, whiria te kaha tūāmanomano.***
>
> *Together we can use our strengths to achieve more. All learning contributes specific threads that we can use to weave a rope strong enough to get us where we want to go, do what we want to do, and be what we want to be.*

The world is full of patterns and structures that we use mathematics and statistics to understand.

Mathematics and statistics enable us to notice, explore, and describe similarities, regularities and irregularities, and trends in the natural, mathematical, technological, and social worlds. They provide tools and ways of working that can reveal patterns and structures useful for decision making, understanding and predicting phenomena, and creating new insights.

> ***Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.***
>
> *Do not catch hold of the loose vine but lay hold of the main vine. You can use the strength of the aka matua (main vine) for the sure footing you will need to reach for new ideas and to climb to new heights.*

The world is characterised by change and variation that we use mathematics and statistics to understand.

The world embodies a multitude of temporary and permanent relationships in which change and variation occur. Some relationships are linear; others are exponential. Mathematics and statistics enable us to systematically describe and analyse different types of change and variation, and to generate insights and make predictions about them.

> ***Ko te pae tawhiti whāia kia tata, ko te pae tata whakamaua kia tina.***
>
> *Seek to bring distant horizons closer and cherish those that you have attained. There will always be pae tawhiti, the 'not yet', but we can move ever closer to that for which we strive.*

Mathematical and statistical logic and reasoning enable us to identify and explain relationships and to justify conclusions.

Reasoning from observation (induction) and reasoning from theory (deduction) allow us to explore situations using mathematics and statistics. Mathematical and statistical logic and reasoning differentiate what is probable from what is possible and allow us to draw reliable conclusions about what is reasonable or not.

Kotahi te kōhao o te ngira e kuhuna ai te miro mā, te miro pango, te miro whero. (Pōtatau Te Wherowhero)

There is but a single eye of the needle through which white, black, and red threads must pass together, yet each thread keeps its own colour and integrity while adding its strength and beauty to the others.

The interface between mātauranga Māori and mātauranga mathematics and statistics offers opportunities for insights that uphold the integrity of each knowledge system.

Mātauranga Māori and mathematics and statistics are different systems for viewing, understanding, and organising the world and for guiding how we operate within it. Mātauranga Māori makes meaningful and distinctive contributions to mathematical inquiry and knowledge in Aotearoa New Zealand, just as mathematical and statistical insights contribute to mātauranga Māori. When considering concepts, processes, and artifacts from te ao Māori, we maintain their integrity by exploring the mātauranga Māori associated with them before formulating mathematical and statistical hypotheses about them.

Nō ngā tūpuna, tuku iho, tuku iho.

The human ideas that have been passed down from generation to generation can help us develop our thinking today.

Mathematics and statistics have a continuous, evolving human history.

Mathematics and statistics have been constructed over thousands of years across the globe as we have grappled with notions of quantity, numerical representation, measurement, dimension, and pattern. They continue to be constructed from ideas drawn from many cultures. In Aotearoa New Zealand, our location in Te Moana-nui-a-Kiwa – with its multiple cultures, artifacts, and knowledges – contributes to mathematics and statistics.

Know

Contexts

Mātauranga tau | Number

Cultures use *Number* to represent, describe, and compare quantities. We operate on these quantities, and use them to estimate, calculate, reason, and justify.

Taurangi | Algebra

Algebra focuses on making and using generalisations to reason mathematically, and on identifying patterns and underlying mathematical relationships. These generalisations, patterns, and relationships can be represented and communicated using diagrams, graphs, and symbols (including variables).

Ine | Measurement

Measurement provides the tools and concepts for quantifying phenomena in the world by estimating, measuring accurately, and using appropriate units, including those from Māori, Pacific, and metric systems of measurement. Many cultures use both standard and non-standard units to measure tangible and intangible quantities.

Mokowā | Space

Space focuses on visualising, representing, and reasoning about the shape, position, orientation, and transformation of objects. It takes account of tools and techniques from the natural world used by many cultures.

Tauanga | Statistics

Statistics focuses on tools, concepts, and systematic process for interpreting situations, using data and its context to understand uncertainty and make predictions. Every piece of data is a taonga to be kept safe and treated ethically and respectfully.

Tūponotanga | Probability

Probability focuses on tools and concepts for quantifying chance, dealing with expectation, and using evidence to identify how likely events are to occur. Probabilistic thinking is evident throughout tikanga and mātauranga Māori.

Do
Practices

Te tūhura pūāhua | Investigating situations

When we investigate situations using mathematics and statistics, we describe and explore them to build our understanding of them. Māori, Pacific, and other world-views may offer us different ways of understanding these situations. When investigating, we need to decide which approaches, concepts, and tools to use and how to use them. We often begin with a question or focus of interest and proceed in systematic but flexible ways, using mathematical and statistical concepts to make sense of solutions, stories about data, and conclusions in context. We conclude by evaluating the investigation, which involves reflecting on the solutions and outcomes and our approaches and choices to determine whether they were reasonable, made sense in context, and could be improved on in future investigations.

Te whakaata pūāhua | Representing situations

When we represent situations mathematically and statistically, we use words or symbols and mental, oral, physical, virtual, graphical, or diagrammatic ways to show concepts and findings. We can use representations to compare, explore, simplify, illustrate, prove, and justify as well as to look for patterns, variations, and trends. They can draw from mātauranga Māori, from Pacific cultures, and from diverse places and periods of history. Representing a situation in multiple ways enables a deeper and more flexible understanding of the situation. It also allows us to communicate with different audiences.

Te tūhono pūāhua | Connecting situations

When we connect situations using mathematics and statistics, we recognise and make links by noticing similarities and differences. Connecting helps us to understand the relationships between concepts, facts, and procedures in mathematics and statistics. This is important because number, algebra, measurement, space, statistics, and probability form a web of interconnected ideas and approaches that can be easier to remember and understand if the connections between them are clear. Connecting also involves linking mathematics and statistics to other learning areas, to mātauranga Māori, and to a range of contexts, including cultural, linguistic, and historical contexts.

Te whakatauwhānui i ngā kitenga | Generalising findings

When we generalise mathematical and statistical findings, we move from specific examples to general principles. We use the patterns, regularities, and structures that we observe to make conjectures that might apply more generally. Further investigation can test and refine these conjectures and determine if they apply in all cases. In statistics, we generalise by using trends and variation in data to make inferences and predictions and to articulate and evaluate claims about similar situations.

Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings

When we explain and justify, we use mathematical and statistical ways of communicating and reasoning to share our ideas and to respond to the ideas, reasoning, and inferences of others. Explaining is how we communicate our inferences and predictions, build arguments, and unpack stories from data. Justifying involves describing why decisions and findings are reasonable, taking into account limitations arising from assumptions and choices and the evidence on which findings are based.

Progress outcome for mathematics and statistics by the end of **year 3**

Understand

I am building knowledge about number, algebra, measurement, space, statistics, and probability, and drawing on the practices of mathematics and statistics.

Through this, I am deepening my understanding that:

> *Whiria te kaha tūātinitini, whiria te kaha tūāmanomano.*

The world is full of patterns and structures that we use mathematics and statistics to understand.

> *Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.*

The world is characterised by change and variation that we use mathematics and statistics to understand.

> *Ko te pae tawhiti whāia kia tata, ko te pae tata whakamaua kia tina.*

Mathematical and statistical logic and reasoning enable us to identify and explain relationships and to justify conclusions.

> *Kotahi te kōhao o te ngira e kuhuna ai te miro mā, te miro pango, te miro whero.*

The interface between mātauranga Māori and mātauranga mathematics and statistics offers opportunities for insights that uphold the integrity of each knowledge system.

> *Nō ngā tūpuna, tuku iho, tuku iho.*

Mathematics and statistics have a continuous, evolving human history.

Know

I know:

Mātauranga tau | Number

In base 10, there are ten digit symbols and their value is defined by their position within a number. Digits in any column are worth ten times as much as those in the column to the right.

Te reo Māori and other Pacific languages explicitly describe the logic of the base 10 numbering system.

Numbers can be partitioned and recombined in different ways by using patterns.

Multiplication and division involve recognising and working with groups, the number of groups, and the total.

Fractions show parts of a whole in a region, a measurement, or a set of objects. The same amount (e.g., a half or a quarter) can be shown by equivalent fractions.

I know how to:

- recognise, read, write, and order whole numbers up to 10,000
- group, partition, and recombine whole numbers up to 1,000
- add and subtract two- and three-digit numbers
- multiply two single-digit numbers or multiply a single-digit and a two-digit number
- divide whole numbers with a single-digit divisor and no remainders
- recognise, read, write, represent, and order halves, thirds, quarters, fifths, sixths, and eighths
- find a unit fraction of a whole (e.g., a region, measurement, or set of objects), and add unit fractions with the same denominator.

Know

Taurangi | Algebra

The commutative property applies to addition (e.g., 2 + 5 = 5 + 2) and multiplication (e.g., 5 × 2 = 2 × 5).

The additive identity is 0 (e.g., 4 + 0 = 4 and 5 – 0 = 5), and the multiplicative identity is 1 (e.g., 5 × 1 = 5 and 4 ÷ 1 = 4).

The equal sign is relational; it shows that the two sides of an equation represent the same quantity.

Patterns can be made of numeric or spatial elements in a sequence governed by a rule.

Identifying the rule of a pattern involves working out the unit of repeat.

An algorithm is a sequence of rules that can be followed.

I know how to:

- recall addition facts to 20 and their corresponding subtraction facts
- recall multiplication and corresponding division facts for twos, fives, and tens
- solve true and false number sentences and open number sentences
- use the additive and multiplicative identities and commutative property
- find another element of a pattern, given part of it
- describe a rule that explains how a pattern works
- follow, and create patterns from, rules or simple algorithms.

Ine | Measurement

Measuring starts at the beginning of the object being measured. The size of the measurement unit must remain the same. Measurement units are repeated with no gaps or overlaps. The measurement is the total number of units used.

Length around the outside of a two-dimensional shape gives perimeter, covering a surface gives area, and filling a three-dimensional shape gives capacity or volume.

I know how to:

- estimate and then reliably measure length, capacity, and mass, using standard metric units
- use rulers, scales, square grids, and cubes to measure
- tell the time to hours, half hours, and quarter past or quarter to the hour, using language and a range of cultural tools, including analogue and digital clocks
- find out how far something has been turned, using half and quarter turns as benchmarks.

Know

Mokowā | Space

Patterns and regularities in shapes can be used to compare, classify, and predict.

Two-dimensional shapes can be composed or decomposed to form new shapes and can have symmetry.

Shapes and objects can flip (reflection), turn (rotation), and slide (translation) and be used to create patterns.

Objects can be rotated in space and may appear different from other perspectives.

Maps are two-dimensional representations of places in the world. They use symbols to show locations and landmarks.

I know how to:

- visualise, identify, compare, and classify two- and three-dimensional shapes
- compose and decompose two-dimensional shapes using the properties of shapes, such as lines of symmetry
- predict and justify what will happen to two-dimensional shapes if you rotate, reflect, or translate them
- use pepeha to describe location by referring to environmental features
- draw simple maps of familiar places to provide directions
- interpret simple maps to locate objects and pathways.

Tauanga | Statistics

Data is information about the world and comes in many forms.

People and the environment are not data, but data can tell us things about people, their lives, and their environment.

Summary investigative questions and the statistical enquiry cycle (PPDAC – Problem, Plan, Data, Analysis, Conclusion) are used to investigate a group.

Data visualisations are representations of all available values of one or more variables that reveal relationships or tell a story.

I know how to:

- pose summary investigative questions about everyday situations, using categorical data and discrete numerical (whole-number) data
- use survey and data-collection questions
- collect, record, and sort data or use secondary data sources
- create and make statements about data visualisations
- answer an investigative question by choosing statements from findings
- identify relevant features in others' data visualisations.

Tūponotanga | Probability

A chance-based situation has a set of possible outcomes that can be arranged into events. The probability of an event is the chance of it occurring.

The statistical enquiry cycle (PPDAC) can be used for chance-based investigations for predicting outcomes of everyday situations and activities and whether they are certain, likely, possible, unlikely, or impossible.

I know how to:

- explore chance-based investigative questions about games and everyday situations in my life
- collect and record data to answer chance-based investigative questions
- create and describe data visualisations for the frequencies of outcomes in chance-based situations
- explain and question statements about chance-based situations, with reference to data.

Do

Te tūhura pūāhua | Investigating situations

I can:
- work with others to pose a question for investigation
- find entry points for addressing a question, identifying relationships and relevant prior experience and knowledge
- work with others to plan an investigation pathway and follow it
- describe progress on the investigation pathway
- work with others to make sense of outcomes or conclusions in light of a given situation and context.

Te whakaata pūāhua | Representing situations

I can:
- use representations to explore, find, and illustrate patterns
- use representations to learn new ideas and explain ideas to others
- select or create appropriate mental, oral, physical, or virtual representations
- use visualisation to mentally represent and manipulate groups and shapes.

Te tūhono pūāhua | Connecting situations

I can:
- suggest connections between ideas and approaches
- suggest connections between different representations
- connect new ideas to things I already know
- make connections with ideas in other learning areas and in familiar local contexts.

Te whakatauwhānui i ngā kitenga | Generalising findings

I can:
- recognise and explore patterns and make conjectures and draw conclusions about them
- identify relationships, including similarities, differences, and new connections
- look for patterns and regularities that might be applied in another situation or always be true
- make and test conjectures, using reasoning and counterexamples to decide if they are true or not
- use words and pictures to express generalisations.

Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings

I can:
- make statements and give explanations about what I notice and wonder
- make statements and give explanations deductively based on prior knowledge
- ask questions to clarify and understand others' thinking
- use evidence and reasoning to explain why I agree or disagree with statements
- develop collective understandings by sharing and building on ideas with others
- present basic explanations and arguments for an idea, solution, or process.

Mathematics and statistics **progress steps**

Progress steps alert teachers to specific aspects of learning that are essential and time-sensitive as students work towards the progress outcome for this phase. They support teachers to notice, recognise, and respond to student learning in a timely fashion, as teachers offer multiple opportunities for learning and practice. These opportunities will be more meaningful for students if they are relevant to their cultural backgrounds and experiences, and if they allow students to use their preferred communication methods, including augmentative and alternative communication (AAC) or assistive technology.

As students move through school, they use mathematics more and more in their daily lives and across all learning areas. Therefore, from year 4 onwards, learning in numeracy is described within each learning area's progress outcomes.

	Mathematics and statistics progress step **during the first six months**
	Building on the foundation from early learning, students can:
Subitising	› recognise instantly the total number of objects in a group of up to six
Number structure	
Operations: Addition and subtraction	› join and separate groups of up to a total of 10 objects, and find the result by grouping and counting
Operations: Multiplication and division	
Rational numbers	
Equality	
Patterns	› copy, continue, create, and describe a repeating pattern with two elements
Measurement	› compare directly two objects by an attribute (e.g., length, mass, capacity)
Classification	› sort shapes and objects by one feature (e.g., colour, shape), identifying the feature chosen
Spatial reasoning	› compose by trial and error an outlined target shape using smaller shapes, and decompose a shape into smaller shapes › follow instructions to move to a familiar location or locate an object.
Variability	

Mathematics and statistics progress step **during the first year**	Mathematics and statistics progress step **during the second year**
Building on their progress from previous months, students can:	***Building on their progress from the previous year, students can:***
› recognise instantly the total number of objects in two patterns, each of up to five objects	› partition a pattern of up to 10 objects, instantly recognise the number of objects in each part, and confirm the total number in the pattern using the parts
› partition and recombine sets of up to 10 in different ways › recognise and represent in different ways, including in te reo Māori, the ten-and-ones structure of teen numbers (11-19)	› group, partition, and recombine whole numbers up to 100
› join and separate groups of up to a total of 20 objects, and find the difference between groups by grouping and counting	› add and subtract numbers up to 100 by grouping and using number patterns
› multiply and divide by making equal groups and using grouping or counting	› multiply and divide by grouping and using number patterns
› recognise, and represent in different ways, halves and quarters of sets and regions	› recognise the relationships between related fractions (e.g., one half is the same as two quarters) › find a half, quarter, or a third of a set by recognising groups and patterns rather than sharing by ones
	› show that in an equation, both sides of the equal sign represent the same quantity
› copy, continue, create, and describe a repeating pattern with three elements, and identify missing elements in a pattern	› use both the unit of repeat and the ordinal position (e.g., first, second, and third) of a repeating pattern to predict further elements
› compare the length, mass, volume, and capacity of objects indirectly (i.e., by comparing each of them with another object)	› estimate, and use a standard informal unit repeatedly to measure, the length, mass, volume, or capacity of an object
› sort and re-sort shapes and objects by features, identifying the feature chosen	
› visualise and anticipate which smaller shapes might compose a target shape, and then check by making the shape › follow and give instructions to move to a familiar location or locate an object.	› visualise and anticipate which smaller shapes might compose or decompose a target shape, and then check by making the shape › follow and give movement instructions that involve familiar reference points, direction, distances (number of steps), and half and quarter turns
	› identify possible outcomes and notice variations in outcomes for familiar activities and situations involving chance.

Progress outcome for mathematics and statistics by the end of **year 6**

Understand

I am building knowledge about number, algebra, measurement, space, statistics, and probability, and drawing on the practices of mathematics and statistics.

Through this, I am deepening my understanding that:

Whiria te kaha tūātinitini, whiria te kaha tūāmanomano.

The world is full of patterns and structures that we use mathematics and statistics to understand.

Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.

The world is characterised by change and variation that we use mathematics and statistics to understand.

Ko te pae tawhiti whāia kia tata, ko te pae tata whakamaua kia tina.

Mathematical and statistical logic and reasoning enable us to identify and explain relationships and to justify conclusions.

Kotahi te kōhao o te ngira e kuhuna ai te miro mā, te miro pango, te miro whero.

The interface between mātauranga Māori and mātauranga mathematics and statistics offers opportunities for insights that uphold the integrity of each knowledge system.

Nō ngā tūpuna, tuku iho, tuku iho.

Mathematics and statistics have a continuous, evolving human history.

Know

I know:

Mātauranga tau | Number

In our number system, each place value is a power of 10, and this continues infinitely.

Multiplication and division problems can involve equal groups, rates, comparisons, combinations, part-whole relationships, areas, and volumes.

Fractions are numbers and can describe a measure, a proportional relationship, or an action on another number.

Fractions express ways of sharing that may be different from those in tikanga and mātauranga Māori.

Decimals are a set of fractions that have powers of 10 as their denominators (e.g., $\frac{7}{10}$ or $\frac{7}{100}$) and that can be written as numbers using a decimal point (e.g., 0.7 or 0.07).

A percentage is a fraction with a denominator of 100 (e.g., $\frac{7}{100}$ is 7%).

I know how to:

- recognise, read, write, order, partition, recombine, and represent whole numbers up to 1,000,000
- add and subtract whole numbers and decimals to two places
- multiply two- and three-digit whole numbers
- divide whole numbers by one- or two-digit divisors
- find factors of numbers up to 100
- recognise, read, write, represent, compare, and order fractions, decimals (to three places), and percentages
- convert between fractions, decimals, and percentages
- find equivalent fractions for halves, thirds, quarters, sixths, and eighths, and represent fractions in their simplest form
- find a simple fraction or percentage of a whole number.

Know

Taurangi | Algebra

The associative property applies to addition and multiplication (e.g., 3 × (2 × 7) = (3 × 2) × 7).

The distributive property applies to multiplication over addition and subtraction (e.g., 3 × (10 + 7) = (3 × 10) + (3 × 7)).

The equal (=) and inequality (<, >) signs show relationships.

In a pattern, the relationship between the ordinal position (e.g., first, second, third) and the corresponding element is useful for finding the pattern rule.

Tables and XY graphs provide a way of organising the positions and elements of a pattern to reveal relationships or rules.

An algorithm is a set of instructions for solving a problem.

I know how to:

- recall multiplication facts to 10 × 10 and corresponding division facts
- use the distributive, commutative, and associative properties
- solve open number sentences and true or false number sentences involving equality or inequality
- use tables, XY graphs, and diagrams to find relationships between elements of growing patterns
- develop a rule in words about a linear pattern
- use a rule to make predictions
- create and use algorithms for making decisions that involve clear choices.

Ine | Measurement

Mātauranga Māori draws on knowledge of te ao tūroa and has meaningful ways of measuring things (e.g., Maramataka).

The metric measurement system is based on powers of ten.

Measurements can contain units and parts of units, and need the unit recorded with the amount (e.g., 1.3 km).

Angles are a measure of turn and can be measured in degrees.

I know how to:

- read measurement tools and interpret scales accurately
- convert between units of time and solve duration-of-time problems
- visualise, estimate, and find the perimeter and area of rectangles and the volume of rectangular prisms
- describe an angle using the benchmarks 90 degrees, 180 degrees, and 360 degrees.

Know

Mokowā | Space

Two- and three-dimensional shapes have consistent properties that can be used to define, compare, classify, predict, and identify relationships between them.

Shapes can be rotated, reflected, translated, and resized.

Viewing objects from different angles gives different perspectives.

Mātauranga Māori often identifies location in the natural world as a form of mapping (e.g., for travelling).

Position can be described using known environmental features and signs from te ao tūroa.

Maps use grid references or coordinates to specify places, scales to show distances, and connections to show pathways.

I know how to:

- classify two-dimensional shapes and prisms using their spatial properties to justify my classifications
- perform and describe rotations, reflections, translations, and resizing on two-dimensional shapes and simple geometric patterns
- visualise and represent three-dimensional shapes from different viewpoints
- visualise and draw nets for rectangular prisms
- use grid references, simple scales, the language of direction (compass points), distance (in m, km), and turn (in degrees) to locate and describe positions and pathways.

Tauanga | Statistics

Data about people and te ao tūroa can have negative impacts so must be collected, used, and stored carefully.

The statistical enquiry cycle (PPDAC – Problem, Plan, Data, Analysis, Conclusion) can be used in summary, comparison, and time-series investigations (e.g., about school-related matters).

Different data visualisations for the same data can lead to different insights.

I know how to:

- pose investigative questions about school contexts for summary, comparison, and time-series situations, and make predictions or assertions about what I expect to find
- plan how to collect primary data or to use provided secondary data
- use and describe a variety of data visualisations, identifying features, patterns, and trends in context and answering the investigative question
- interrogate others' survey or data-collection questions, and identify and explain features and errors in others' data visualisations and statements about data.

Tūponotanga | Probability

The statistical enquiry cycle (PPDAC) can be used for chance-based investigations.

Probabilities and the language of probability are associated with values between 0 or 0% (impossible) and 1 or 100% (certain).

A probability experiment involves repeated trials. Results may vary in trials. The experimental probability of an event is the number of times the event occurs divided by the total number of trials.

I know how to:

- pose investigative questions for a chance-based situation with equally likely outcomes, listing all possible outcomes for the situation
- plan, conduct, and record data for a probability experiment
- create and describe data visualisations for the distribution of observed outcomes from a probability experiment, using them to answer the investigative question
- compare my findings with those of others when undertaking probability experiments.

Do

Te tūhura pūāhua | Investigating situations

I can:

- pose a question for investigation
- find entry points for addressing a question, identifying relevant prior knowledge, givens, and relationships
- plan an investigation pathway and follow it step by step
- monitor and evaluate progress, adjusting the investigation pathway if necessary
- make sense of outcomes or conclusions in light of a given situation and context.

Te whakaata pūāhua | Representing situations

I can:

- use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns and variations
- use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments
- select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations
- use visualisation to mentally represent and manipulate objects and ideas.

Te tūhono pūāhua | Connecting situations

I can:

- suggest connections between ideas and approaches
- suggest connections between different representations
- connect new ideas to things I already know
- make connections with ideas in other learning areas and in familiar cultural, linguistic, and historical contexts.

Te whakatauwhānui i ngā kitenga | Generalising findings

I can:

- recognise and explore patterns, and make conjectures and draw conclusions about them
- identify relationships, including similarities, differences, and new connections
- look for patterns and regularities that might be applied in another situation or always be true
- make and test conjectures, using reasoning and counterexamples to decide if they are true or not
- use appropriate symbols to express generalisations.

Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings

I can:

- make statements and give explanations inductively based on observations or data
- make statements and give explanations deductively based on knowledge, definitions, and rules
- critically reflect on others’ thinking, evaluating their logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements
- develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others
- present reasoned explanations and arguments for an idea, solution, or process.

Mathematics and statistics **progress steps**

Progress steps alert teachers to specific aspects of learning that are essential and time-sensitive as students work towards the progress outcome for this phase. They support teachers to notice, recognise, and respond to student learning in a timely fashion, as teachers offer multiple opportunities for learning and practice. These opportunities will be more meaningful for students if they are relevant to their cultural backgrounds and experiences, and if they allow students to use their preferred communication methods, including augmentative and alternative communication (AAC) or assistive technology.

	Mathematics and statistics progress step **during year 4** *Building on their progress from the previous year, students can:*
Number structure	› recognise, read, write, order, partition, recombine, and represent whole numbers up to 10,000
Operations: Addition and subtraction	› use their recalled addition and subtraction basic facts to solve problems › add and subtract two- and three-digit numbers reliably and efficiently
Operations: Multiplication and division	› use the relationship between multiplication and division to divide › recall multiplication and corresponding division facts for threes and fours
Rational numbers	› represent common fractions, including those greater than 1, on a number line
Equality	› solve addition and subtraction open number sentences using the relationship between the two sides of the equal sign
Spatial reasoning	› visualise, predict, and identify a shape that is a reflection, rotation, or translation of a given two-dimensional shape.
Variability	

Mathematics and statistics progress step **during year 5**

Building on their progress from the previous year, students can:

- recognise, read, write, order, partition, recombine, and represent whole numbers up to 100,000

- add or subtract any whole number reliably and efficiently

- multiply two-digit numbers reliably and efficiently using the distributive property
- recall multiplication and corresponding division facts for sixes, eights, and nines

- compare fractions with a benchmark fraction and put them in order
- convert between benchmark fractions, decimals, and percentages (e.g., $\frac{1}{2}$ = 0.5 = 50%)
- represent decimals, fractions, and percentages using both discrete and continuous models

- solve open number sentences involving all operations using the relationship between the two sides of the equal sign

- visualise and draw nets for a cube

- recognise the need for relevant and usable data to answer investigative questions
- suggest reasons why data may vary in a familiar context.

Phase 3 (years 7–8)

Progress outcome for mathematics and statistics by the end of **year 8**

Understand

I am building knowledge about number, algebra, measurement, space, statistics, and probability, and drawing on the practices of mathematics and statistics.

Through this, I am deepening my understanding that:

Whiria te kaha tūātinitini, whiria te kaha tūāmanomano.

The world is full of patterns and structures that we use mathematics and statistics to understand.

Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.

The world is characterised by change and variation that we use mathematics and statistics to understand.

Ko te pae tawhiti whāia kia tata, ko te pae tata whakamaua kia tina.

Mathematical and statistical logic and reasoning enable us to identify and explain relationships and to justify conclusions.

Kotahi te kōhao o te ngira e kuhuna ai te miro mā, te miro pango, te miro whero.

The interface between mātauranga Māori and mātauranga mathematics and statistics offers opportunities for insights that uphold the integrity of each knowledge system.

Nō ngā tūpuna, tuku iho, tuku iho.

Mathematics and statistics have a continuous, evolving human history.

Know

I know:

Mātauranga tau | Number

Decimals continue the place-value system using negative powers of ten.

Multiplying a positive number by a number between 0 and 1 results in an answer smaller than the original number.

Division can result in a remainder expressed as a whole number, fraction, or decimal.

On a number line, fractions and decimals occur between integers, and negative numbers are to the left of 0.

Positive and negative numbers can be added and subtracted.

I know how to:

- represent whole numbers and decimals using powers of ten
- divide whole numbers reliably and efficiently
- recognise, read, write, represent, compare, order, and convert between fractions, decimals, and percentages
- add and subtract decimals to three places
- add and subtract fractions with related denominators
- multiply fractions and decimals by whole numbers
- represent fractions in their simplest form
- add and subtract integers.

Know

Taurangi | Algebra

The inverse property applies to addition (e.g., $3 + -3 = 0$) and multiplication (e.g., $3 \times \frac{1}{3} = 1$).

The commutative, associative, distributive, and identity properties work the same for all numbers.

A variable can be used to represent any number.

Functions are relationships or rules where each member of an input set associates with a single output.

Linear patterns and functions have a constant rate of change. They can be represented by ordered pairs, tables, XY graphs, and a rule (equation).

Algorithms help solve problems in a systematic way. Their instructions are created, tested, and revised.

I know how to:

- identify and describe the properties of prime, composite, square, and cube numbers and the divisibility rules for 2, 3, 5, 9, and 10
- use words and symbols to describe and represent the properties of operations (commutative, distributive, associative, inverse, and identity)
- solve linear equations by trial and improvement and by applying inverse operations
- use variables to represent a rule about a linear pattern, and use the rule to make predictions
- represent and connect linear functions using tables, equations, and XY graphs
- create, test, revise, and use algorithms to identify, interpret, and explain patterns.

Ine | Measurement

In the metric system, there are base measurements with prefixes added to show the size of units.

Metric measurements can be converted from fractions to whole numbers, and vice versa, by changing units.

Shapes can be decomposed or recomposed to help us find perimeters, areas, and volumes.

When two line segments meet they form an angle, which can be thought of as a rotation of one of the line segments.

I know how to:

- estimate and then measure length, area, volume, capacity, mass, temperature, data storage, time, and angle, using appropriate metric units
- convert between measurement units
- read analogue and digital measurement tools, round appropriately, and interpret scales accurately
- visualise, estimate, and find the perimeter and area of shapes composed of triangles and rectangles
- read, interpret, and use timetables and charts that present measurement information.

Know

Mokowā | Space

Spatial properties of simple polygons and polyhedra can also apply to more complex two- and three-dimensional shapes.

Three-dimensional shapes can be represented by two-dimensional images.

The invariant properties of two- and three-dimensional shapes do not change under different transformations.

Position, direction, and pathways can be described using te ao tūroa, as in Māori and Pacific systems of knowledge, or using scale, compass points, and environmental features.

Coordinate systems and maps can express position, direction, and pathways.

I know how to:

- classify shapes based on their geometric properties
- visualise and draw nets for prisms that have a fixed cross section
- use plan-view drawings to visualise and construct three-dimensional shapes
- find unknown angles and identify angle properties of intersecting lines
- make combinations of transformations that use the invariant properties of shapes
- use scale, compass points, and coordinate systems to interpret and describe positions and pathways.

Tauanga | Statistics

Datasets have a whakapapa. How and why data about people and te ao tūroa is collected, interpreted, and stored needs to benefit, include, and protect them, and must not harm them. People need to know who they are giving data to and why before they agree to contribute to a dataset.

The statistical enquiry cycle (PPDAC – Problem, Plan, Data, Analysis, Conclusion) can be used to conduct data-based investigations about the wider community.

Data visualisations show patterns, trends, and variations. Alternative visualisations of the same data can lead to different insights and communicate different information.

I know how to:

- pose investigative questions about local rohe and community matters and make predictions or assertions about what I expect to find
- determine the variables needed to answer investigative questions, and plan how to collect data for each variable
- collect data from a group (when all of the group can be surveyed), or source and use data collected by others
- analyse data and communicate findings in context
- examine the data-collection methods, data visualisations, and findings of others' statistical investigations to see if their claims are believable and reasonable.

Tūponotanga | Probability

The statistical enquiry cycle (PPDAC) can be used for chance-based investigations, using sampling with replacement.

In a probability experiment with independent trials, results from one trial do not affect results from other trials.

Estimated probabilities from experiments and theoretical model probabilities will differ.

If all possible outcomes in a chance-based situation are equally likely, the probability of an event = $\frac{\text{the number of ways the event can happen}}{\text{the total number of possible outcomes}}$.

I know how to:

- recognise claims or misconceptions in relation to chance-based situations
- pose investigative questions for chance-based situations, including those with not equally likely outcomes
- plan, conduct, and systematically record data from probability experiments
- use data visualisations to describe the distribution of observed outcomes from probability experiments and possible outcomes for theoretical probability models
- agree or disagree with others' conclusions by interrogating their chance-based investigations.

Do

Te tūhura pūāhua | Investigating situations

I can:

- pose a question for investigation
- find entry points for addressing a question, identifying relevant prior knowledge, givens, assumptions, and relationships
- plan an investigation pathway and follow it in an organised way
- monitor and evaluate progress, adjusting the investigation pathway if necessary
- make sense of outcomes or conclusions in light of a given situation and context.

Te whakaata pūāhua | Representing situations

I can:

- use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns, variations, and trends
- use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments
- select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations
- use visualisation to mentally represent and manipulate relationships, objects, and ideas.

Te tūhono pūāhua | Connecting situations

I can:

- suggest connections between ideas and approaches
- suggest connections between different representations
- connect new ideas to things I already know
- make connections to ideas in other learning areas and in diverse cultural, linguistic, and historical contexts.

Te whakatauwhānui i ngā kitenga | Generalising findings

I can:

- recognise and explore patterns, and make conjectures and draw conclusions about them
- identify relationships, including similarities, differences, and new connections
- look for patterns and regularities that can be applied in another situation or are always true
- make and test conjectures, using reasoning and counterexamples to decide if they are true or not
- use appropriate symbols to express generalisations.

Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings

I can:

- make statements and give explanations inductively based on observations or data
- make statements and give explanations deductively based on knowledge, definitions, and rules
- critically reflect on others' thinking, distinguishing between correct and flawed logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements
- develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others
- present reasoned, coherent explanations and arguments for an idea, solution, or process.

Phase 4
(years 9–10)

Progress outcome for mathematics and statistics by the end of **year 10**

Understand

I am building knowledge about number, algebra, measurement, space, statistics, and probability, and drawing on the practices of mathematics and statistics.

Through this, I am deepening my understanding that:

Whiria te kaha tūātinitini, whiria te kaha tūāmanomano.

The world is full of patterns and structures that we use mathematics and statistics to understand.

Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.

The world is characterised by change and variation that we use mathematics and statistics to understand.

Ko te pae tawhiti whāia kia tata, ko te pae tata whakamaua kia tina.

Mathematical and statistical logic and reasoning enable us to identify and explain relationships and to justify conclusions.

Kotahi te kōhao o te ngira e kuhuna ai te miro mā, te miro pango, te miro whero.

The interface between mātauranga Māori and mātauranga mathematics and statistics offers opportunities for insights that uphold the integrity of each knowledge system.

Nō ngā tūpuna, tuku iho, tuku iho.

Mathematics and statistics have a continuous, evolving human history.

Know

I know:

Mātauranga tau | Number

Decimals can be terminating, repeating and infinite, or non-repeating and infinite (irrational numbers).

There are an infinite number of rational numbers between any two numbers.

Multiplying a fraction by an equivalent form of 1 (e.g., $\frac{3}{3}$) results in an equivalent fraction.

For positive numbers, dividing by a divisor between 0 and 1 gives a result bigger than the dividend.

A rate compares two quantities that have different units of measure. A ratio is a comparison of two like quantities.

I know how to:

- add and subtract fractions with different denominators by using equivalent fractions
- multiply and divide two integers, two fractions, or two decimals
- find fractions or percentages of a number
- compare fractions that arise from division-as-sharing (e.g., sharing 2 among 5 vs sharing 3 among 7)
- simplify fractions using the highest common factor
- use rates to model and represent change
- use and apply ratios to model everyday situations.

Taurangi | Algebra

The properties of operations (commutative, distributive, associative, inverse, and identity) apply to numbers and variables.

There is an order of operations when using numbers and variables.

Functions can be expressed as algebraic equations, XY graphs, tables, or in words.

I know how to:

- operate on numbers with whole-number exponents and develop the rule for multiplying or dividing numbers with exponents and the same base
- express functions arising from linear and simple quadratic patterns
- graph linear functions and interpret the gradient, x-intercept, and y-intercept in relation to the function or the practical situation represented

Know

Taurangi | Algebra (cont.)

There are many different, equivalent equations for expressing a linear function.

Algorithms can be efficient or inefficient. More efficient algorithms have fewer steps.

I know how to:

- substitute into, rearrange, and simplify expressions, combining like terms as needed
- create or use a formula, rule, equation, or inequality, solve for unknowns, and evaluate by substitution
- identify how sequence, selection, and iteration are used in algorithms for generating patterns.

Ine | Measurement

Decimal measures are used for very small durations (milliseconds); the rest of time measurement uses a different system (based on 12 and 60).

The defining characteristics of prisms are used to describe what they are and how they are formed.

Resizing a shape changes its perimeter, area, and volume.

I know how to:

- estimate, calculate, and represent accurately measurements using significant figures
- derive and use the formula for the perimeter or circumference and area of polygons and circles
- find the surface area and volume or capacity of prisms and cylinders
- scale a shape by a factor and derive the scale factor for the scaled shape's area or volume.

Mokowā | Space

For all polygons, there is a generalisation for the sum of interior angles and the sum of exterior angles.

Angles between parallel lines and a transversal have known relationships (corresponding, alternate, or co-interior angles).

In similar shapes, corresponding angles are equal and lengths of corresponding sides are proportional.

In right-angled triangles, there is a fixed relationship between the lengths of the three sides given by Pythagoras' theorem.

A point has zero dimensions, a line has one dimension, a plane is two-dimensional, and a solid is three-dimensional. In mathematics, there can be more than three dimensions.

I know how to:

- reason about unknown angles in situations involving parallel lines and transversals and the interior and exterior angles of polygons
- use the properties of similarity in two-dimensional shapes, including right-angled triangles, to find unknown lengths
- use and apply Pythagoras' theorem to find the length of an unknown side in a right-angled triangle
- use invariant properties to transform a set of points in the XY plane by translation, reflection about an axis, and rotation about a given point by a multiple of 90 degrees
- represent three-dimensional shapes with two-dimensional drawings and digital tools.

Know

Tauanga | Statistics

Māori have rangatiratanga over their data, including inherent rights and interests in relation to the collection, use, storage, and ownership of it.

People have rights and obligations in relation to their own data and that of others. Different countries have different laws about data and privacy.

The statistical enquiry cycle (PPDAC) can be used to conduct data-based investigations that involve sampling from populations. When sampling from a population, the distribution for a variable varies from sample to sample.

New variables can be created by combining and modifying existing variables.

I know how to:

- pose summary and comparison investigative questions about populations, and investigative questions for time-series and relationship data and for experiments
- plan to collect data for observational studies and experiments, including selecting valid and reliable measurements for variables or sourcing existing datasets
- recategorise variables if needed, and use multiple representations to analyse and visualise data
- communicate findings using evidence from analysis, provide possible explanations for findings, and reflect on predictions or assertions
- critique the findings and claims of others by interrogating all phases of the statistical enquiry cycle.

Tūponotanga | Probability

The statistical enquiry cycle (PPDAC) can be used to conduct chance-based investigations involving simulations.

Theoretical model probabilities and experimental estimates of probabilities are approximations of the true probabilities, which are never known.

In two-stage chance-based situations, the stages can be dependent (leading to conditional probabilities) or independent.

I know how to:

- recognise and pose investigative questions, anticipate what will happen, and state assumptions
- plan probability experiments that use real data to create probability distributions for numerical variables, run simulations, and record data
- describe probability distributions including those involving simple, joint, and conditional probabilities
- identify why claims about chance-based situations might not be valid and consider improvements to how the claim was investigated.

Do

Te tūhura pūāhua | Investigating situations

I can:

- pose a question for investigation
- find entry points for addressing a question, identifying relevant prior knowledge, givens, assumptions, constraints, and relationships
- plan an investigation pathway and follow it in a systematic and organised way
- monitor and evaluate progress, adjusting the investigation pathway if needed
- make sense of outcomes or conclusions in light of a given situation and context.

Te whakaata pūāhua | Representing situations

I can:

- use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns, variations, and trends
- use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments
- select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations
- use visualisation to mentally represent and manipulate relationships, objects, and ideas.

Te tūhono pūāhua | Connecting situations

I can:

- suggest connections between ideas and approaches
- suggest connections between different representations
- connect new ideas to things I already know
- make connections to ideas in other learning areas and in a range of cultural, linguistic, and historical contexts.

Te whakatauwhānui i ngā kitenga | Generalising findings

I can:

- recognise and explore patterns, and make conjectures and draw conclusions about them
- identify relationships, including similarities, differences, and new connections
- look for patterns and regularities that can be applied in another situation or are always true
- make and test conjectures, using reasoning and counterexamples to decide if they are true or not
- use appropriate symbols to express generalisations.

Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings

I can:

- make statements and give explanations inductively based on observations or data
- make statements and give explanations deductively based on knowledge, definitions, and rules
- critically reflect on others' thinking, distinguishing between correct and flawed logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements
- develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others
- present reasoned, coherent explanations and arguments for an idea, solution, or process.

Phase 5
(years 11–13)

Progress outcome for mathematics and statistics by the end of **year 13**

Understand

I am building knowledge about number, algebra, measurement, space, statistics, and probability, and drawing on the practices of mathematics and statistics.

Through this, I am deepening my understanding that:

Whiria te kaha tūātinitini, whiria te kaha tūāmanomano.

The world is full of patterns and structures that we use mathematics and statistics to understand.

Kei hopu tōu ringa ki te aka tāepa, engari kia mau ki te aka matua.

The world is characterised by change and variation that we use mathematics and statistics to understand.

Ko te pae tawhiti whāia kia tata, ko te pae tata whakamaua kia tina.

Mathematical and statistical logic and reasoning enable us to identify and explain relationships and to justify conclusions.

Kotahi te kōhao o te ngira e kuhuna ai te miro mā, te miro pango, te miro whero.

The interface between mātauranga Māori and mātauranga mathematics and statistics offers opportunities for insights that uphold the integrity of each knowledge system.

Nō ngā tūpuna, tuku iho, tuku iho.

Mathematics and statistics have a continuous, evolving human history.

Know

I know:

Mātauranga tau | Number

Geometric sequences can be generated using recursion, and they can help model exponential growth and decay.

The square root of negative one ($\sqrt{-1}$) is represented by i.

Sums or series can be represented using sigma notation.

I know how to:

- find square roots of negative numbers
- calculate the number of possible arrangements in a set, both when the order of the arrangements does matter and when the order does not matter.

Taurangi | Algebra

An equation can undergo a variety of transformations with predictable effects on the solution.

A formula can include multiple parameters and variables, which are represented in different ways depending on the context in which the formula is being used.

Functions can be combined to create new functions by operating on them, including through function composition.

The derivative of a function can be interpreted as its rate of change.

The indefinite integral is a general antiderivative; the derivative of the indefinite integral is the original function.

I know how to:

- solve algebraic equations using the zero-product property (where several expressions multiply to be zero, then one of them must be zero)
- determine the nature of the roots of a quadratic equation
- operate on algebraic fractions
- algebraically solve systems of three linear equations in three-dimensional space, and geometrically interpret the solutions
- predict the effect on the representations of a function when it undergoes a transformation
- use the chain rule to differentiate composite functions.

Know

Ine | Measurement

A point on a unit circle at an angle of θ with the positive x-axis (in standard position) is represented by the coordinates (cos θ, sin θ). This demonstrates the periodic and symmetric nature of the sine and cosine functions, visually and algebraically.

The definite integral gives the signed area bounded between the x-axis and a curve over an interval.

I know how to:

- use simple trigonometric identities (e.g., $\sin^2(\theta) + \cos^2(\theta) = 1$) to simplify calculations
- apply calculus to trigonometric functions
- approximate the area under a curve using rectangles or trapeziums, and improve the approximation
- use derivatives and integrals to solve kinematic problems involving displacement, velocity, and acceleration.

Mokowā | Space

At a single point, the derivative of a function equals the gradient of the tangent line. This can be interpreted as the rate of change of the function at that point.

Tangent lines are local approximations of a function. Near a specific point, the tangent line and the function have approximately the same graph.

The gradient of the tangent line at a local maximum or minimum of a function is 0 or undefined.

I know how to:

- find and graph tangents of a function
- describe curves and circles using parametric equations
- explore and prove conjectures about functions (e.g., about the nature of their graphs, their rates of change, and the area under their curves).

Know

Tauanga | Statistics

Sourcing data and learning from it are at the heart of evidence-based decision making. Data-based information is used to inform and influence decisions, behaviours, policies, and opinions.

How we collect data affects the applicability, quality, diversity, and quantity of the data, as well as the conclusions we draw from it. We need to take care about sources of bias.

For statistical inference, data is sometimes obtained using random sampling to enable and justify sample-to-population inferences. Sometimes experiments are used to justify cause-and-effect inferences.

Randomisation tests are simulation methods that can be used with experiments to assess the strength of evidence for the existence of treatment differences.

Bootstrap confidence intervals are simulation methods that can be used to allow for uncertainties in estimation due to sampling error.

Data can be used to develop and train algorithmic predictive-models. Predictions from these models have a distribution that changes as the conditioning or predictor variables are changed.

I know how to:

- use ethical and responsible data practices when designing and conducting studies, storing and transferring data, and evaluating studies
- determine whether a random sample, an experiment, a secondary data source, or another approach is most suitable for answering an investigative question
- design and implement a simple randomised experiment to compare two groups
- create, manipulate, and merge data from a variety of sources manually and using statistical software, creating a range of relevant data visualisations for it
- select, use, and evaluate appropriate statistical models for making predictions (including regression, time-series, and classification models)
- use the results from analyses to form and communicate conclusions, acknowledge uncertainty, and make new conjectures
- critique data- and chance-based information, data visualisations, embedded statistics, and claims from a variety of sources, including the media.

Tūponotanga | Probability

There are situations in life that involve uncertainty. Probability can help us think about these situations and make decisions on them.

Probabilities can be estimated from gathered (empirical) data or from theoretical models; both approaches make assumptions that might not be valid.

Generating data from a probability model through simulation can demonstrate what outcomes are likely or unlikely under certain conditions, as well as the variability of the outcomes.

Uniform, binomial, Poisson, and normal probability distributions have recognisable key features and can be used to model situations.

I know how to:

- conduct large-scale simulations to model probability outcomes
- predict and explain effects caused by changing the parameters of probability distributions
- estimate and calculate probabilities of independent, combined, and conditional events
- calculate, interpret, and evaluate risk
- calculate, interpret, and evaluate the expected value of a numerical random variable
- make and critique claims for reports on polls by taking into account the informal margin of error and considering possible sources of bias.

Do

Te tūhura pūāhua | Investigating situations

I can:

- pose a question for investigation
- find entry points for addressing a question, identifying relevant prior knowledge, givens, assumptions, constraints, relationships, and concepts
- plan an investigation pathway and follow it in a systematic and organised way
- monitor and evaluate progress, adjusting the investigation pathway if needed
- make sense of outcomes or conclusions in light of a given situation and context.

Te whakaata pūāhua | Representing situations

I can:

- use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns, variations, and trends
- use representations to learn new ideas, explain ideas to others, investigate conjectures, and support arguments
- select, create, or adapt appropriate mental, oral, physical, virtual, graphical, or diagrammatic representations
- use visualisation to mentally represent and manipulate relationships, objects, and ideas.

Te tūhono pūāhua | Connecting situations

I can:

- suggest connections between ideas and approaches
- suggest connections between different representations
- connect new ideas to things I already know
- make connections to ideas in other learning areas and in a range of cultural, linguistic, and historical contexts.

Te whakatauwhānui i ngā kitenga | Generalising findings

I can:

- recognise and explore patterns, and make conjectures and draw conclusions about them
- identify relationships, including similarities, differences, and new connections
- look for patterns and regularities that can be applied in another situation or are always true
- make and test conjectures, using reasoning and counterexamples to decide if they are true or not
- use appropriate symbols to express generalisations.

Te whakamārama me te parahau i ngā kitenga | Explaining and justifying findings

I can:

- make statements and give explanations inductively based on observations or data
- make statements and give explanations deductively based on knowledge, definitions, and rules
- critically reflect on others' thinking, distinguishing between correct and flawed logic and asking questions to clarify and understand
- use evidence, reasoning, and proofs to explain why I agree or disagree with statements
- develop collective understandings by sharing, comparing, contrasting, critiquing, and building on ideas with others
- present reasoned, coherent explanations and arguments for an idea, solution, or process.

At years 11–13, learning in mathematics and statistics becomes increasingly specialised. In addition, all students have opportunities to further develop and use what they have learnt in years 1–10 in a range of approved NCEA subjects.